

Threading of unconcatenated ring polymers at high concentrations: double-folded *vs.* time-equilibrated structures

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Abstract

Unconcatenated ring polymers in concentrated solutions and melt are remarkably well described as double-folded conformations on randomly branched primitive trees. This picture though contrasts recent evidence for extensive intermingling between close-by rings in the form of long-lived topological constraints or threadings. Here, we employ the concept of ring minimal surface to quantify the extent of threadings in polymer solutions of the double-folded rings *vs.* rings in equilibrated Molecular Dynamics computer simulations. Our results show that, the double-folded ring polymers are significantly less threading compared to their counterparts at equilibrium. Second, threadings form through a slow process whose characteristic time-scale is in the order of that for rings random diffusion in solution. These findings are robust, being based on universal (model-independent) observables as the average fraction of threaded length or the total penetrations between close-by rings and the corresponding distribution functions.

Concentrated solutions and melts of unconcatenated and unknotted ring polymers have stimulated intensive theoretical¹⁻²⁰ and experimental²¹⁻²⁷ work in latest years.

Under high concentrations rings challenge most of the peculiarities characterizing the more familiar case of solutions of *linear* chains. First, spatial constraints arising from global topological invariance and the consequential departure⁴ from the Flory-like mechanism²⁸⁻³⁰ for compensation of excluded volume effects imply that the average ring size or gyration radius, R_g , scales in the limit of large polymer mass or contour length, L_c , like^{11,12} $R_g \sim L_c^{1/3}$ while for linear chains²⁸⁻³⁰ $R_g \sim L_c^{1/2}$. Second, absence of free ends implies that rings do not relax *via* common reptation which is, instead, the dominating mechanism for linear chains.²⁸⁻³⁰ Consequently, stress relaxation in ring solutions decays as a power-law²¹ with no sign of the rubber-like plateau of linear melts.^{29,30}

Substantial theoretical progress was made back in the '80s, when Khokhlov and Nechaev¹ and Rubinstein³¹ mapped the problem of rings in entangled solutions to the one of a single ring in an array of fixed obstacles to which it is not topologically linked. In the latter conditions, rings should adopt double-folded conformations on randomly branching primitive trees.^{1,31} Recently,¹¹ explicit numerical mapping of ring polymers in solution to randomly branched structures has demonstrated that relevant properties as the polymer gyration radius or contact frequencies can be accurately reproduced. Further theoretical and numerical investigations^{12,15,18,32} also support the “rings/branched polymers” analogy.

This successful picture is challenged in recent works³³ showing that mutually exposed surfaces between neighbor rings form long-lived topological constraints, commonly known as *threadings*.^{17,20,34,35} Absent in systems of linear chains, threadings are responsible for the observed glassy behaviour of ring solutions under pinning perturbations.^{17,20,36} Conversely, being relaxed only up to the entanglement scale (Section IA in SI), ring polymers folding into branched structures display little interpenetration with close-by neighbors.

To shed light on this apparent conflict, in this Letter we quantify the extent of threadings between distinct pairs of unconcatenated rings in solution and melt by employing the concept

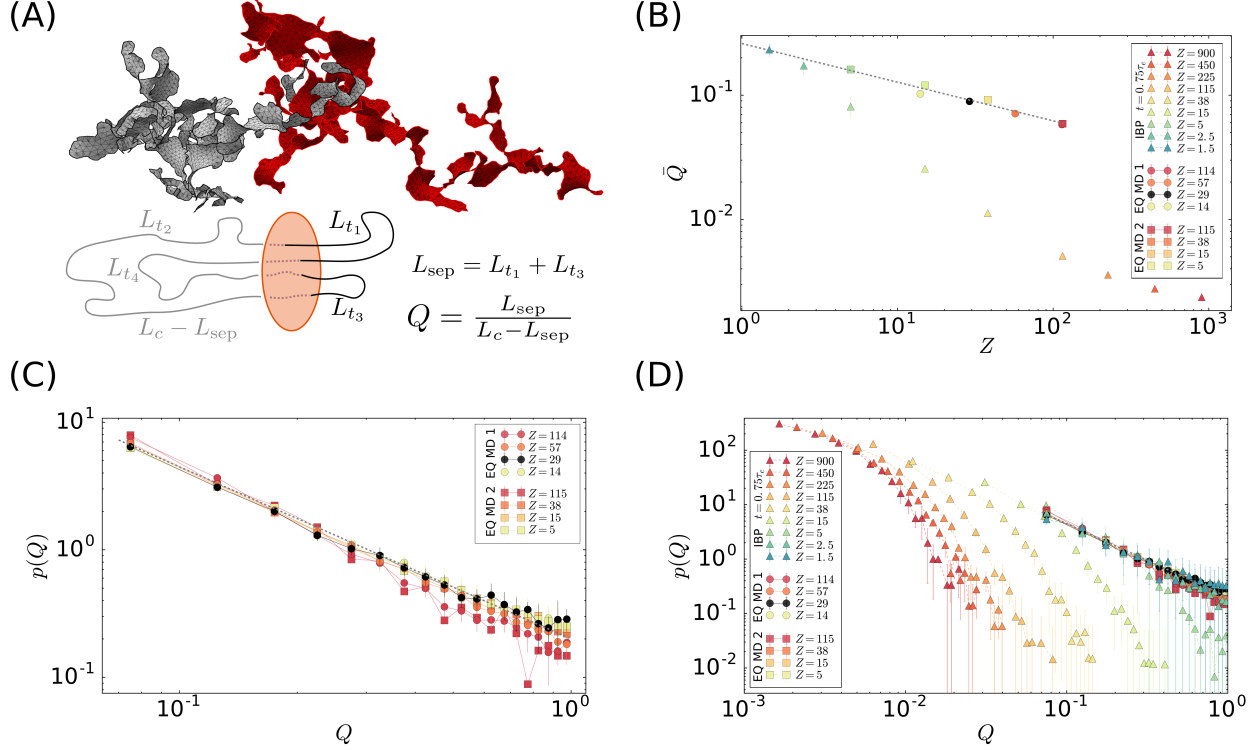


Figure 1: Threading statistics in terms of relative contour length fraction Q . (A) Top: Minimal surfaces of a pair of close-by rings modeled as double-folded polymers on interacting branched primitive trees (IBP-model). Bottom: Schematic representation of one ring (black and grey) penetrating the minimal surface of another ring (orange) of total contour length L_c . L_{t_i} is the contour length of subchain i penetrating the second ring. In this example, four surface penetrations ($n_p = 4$) split the penetrating ring into the segment pairs (L_{t_1}, L_{t_3}) and (L_{t_2}, L_{t_4}) which are on opposite sides of the surface: this defines the separation length, L_{sep} , and its complementary, $L_c - L_{\text{sep}}$. Adapted with permission.³³ (B) Mean relative contour length fraction, \bar{Q} , of one ring threading another ring as a function of ring mass, Z . The dashed line is the best fit to the data for MD-equilibrated rings, $\bar{Q} \approx 0.26Z^{-0.31}$. (C) Probability distribution functions, $p(Q)$ (log-log scale). Results for MD-equilibrated rings from polymer models EQ MD 1 and EQ MD 2. The dashed grey line $p(Q) \sim Q^{-1.35}$ is the best fit to the distributions tail. (D) Comparison between $p(Q)$'s for the IBP-model and MD-equilibrated rings. In panels (C) and (D), the bin size is $= Q_{\text{max}}/20$ with Q_{max} the largest value of Q in the given data set.

of ring *minimal surface* (Fig. 1(A), top), which was recently^{10,33} applied to detect threadings in melts of rings. Specifically, a ring is defined as “threaded” by another ring if its minimal surface is crossed by the other ring (Fig. 1(A), bottom). Here we investigate only two-ring threadings, therefore self-threadings are ignored. Numerical construction of minimal surfaces has been performed in turn for: (1) double-folded ring polymers on interacting randomly branched primitive trees¹¹ (IBP-model) (for details, see Sec. IA in Supporting Information (SI)) and (2) rings in solutions equilibrated through large-scale, brute force Molecular Dynamics (MD) computer simulations. As for the latter, two microscopic, distinct polymer models have been chosen (Sec. IB in SI): (a) The classical Kremer-Grest (KG) polymer model (hereafter, EQ MD 1) from Ref.,⁷ at melt conditions; (b) The generalized KG polymer model (hereafter, EQ MD 2) from Refs.^{11,37} with larger stiffness at semi-dilute conditions. The initial ring conformations adopted in this second case come from the IBP-model, thus we will use the full MD trajectories to characterize the time progression of the threading statistics. To analyze results from the two different polymer models on equal footing, observables will be given as functions of the total number of *entanglements* $Z \equiv L_c/L_e$,^{38,39} where L_c is the ring contour length and L_e is the entanglement length.²⁹ The largest rings which can be equilibrated in reasonable computational time are for $Z \approx 100$ for both set-up’s (see Table S1 in SI for details on the systems and corresponding statistics used). To speed-up the equilibration of the longest rings of EQ MD 1, we used a novel anisotropic doubling scheme (see Sec. IB in SI).

Threadings statistics – Minimal surfaces spanned on the ring polymers are obtained by a slightly modified version of the minimization algorithm from³³ (see Sec. IC in SI). The algorithm is based on successive iterations of triangulations evolving under surface tension by moving the free vertices. Typically, each ring penetrates the minimal surfaces of more of its neighbors and this number grows with Z (see Sec. IE in SI for details). Then, following³³

we introduce the *separation length*:

$$L_{\text{sep}} = \min \left(\sum_{i=\text{even}} L_{t_i}, \sum_{i=\text{odd}} L_{t_i} \right), \quad (1)$$

where L_{t_i} is the (threading) length between the i -th and the $i + 1$ -th penetrations of the surface (Fig. 1(A), bottom). L_{sep} characterizes how much material of the *penetrating* ring is on one side of the *penetrated* ring (the contour length on the other side being $= L_c - L_{\text{sep}}$, of course). Accordingly, the quantity $Q \equiv \frac{L_{\text{sep}}}{L_c - L_{\text{sep}}}$ accounts for the relative extent of contour length on one side with respect to the other, $Q = 1$ meaning the penetrating ring is half split by the penetrated surface.

The mean value $\bar{Q} = \bar{Q}(Z)$, obtained by averaging Q over $\mathcal{O}(10^3)$ up to $\mathcal{O}(10^4)$ interpenetrating rings pairs (see Table S1 in SI), is plotted in Fig. 1(B). Remarkably, data for MD-equilibrated rings collapse on the same (universal) curve characterized by simple power-law decay $\bar{Q} = (0.26 \pm 0.09)Z^{-0.31 \pm 0.09}$ (dashed line). As the two polymer models EQ MD 1 and 2 have different monomer densities and entanglement lengths (Sec. IB in SI) this is a non-trivial result, which pinpoints \bar{Q} as a suitable “order parameter” for characterizing the total extent of threading between close-by rings. In fact, double-folded rings display smaller values for \bar{Q} suggesting a lesser extent of threadings between close-by rings. Fig. 1(C) shows the complete distribution functions, $p(Q)$, for MD-equilibrated rings at different Z 's. Mirroring corresponding averages in Fig. 1(B), $p(Q)$'s from the two different polymer models agree well and the observed power-law behavior $p(Q) \sim Q^{-1.35}$ for $0.1 \lesssim Q \lesssim 1$ agrees with the reported³³ decay for distribution functions, $p(L_{\text{sep}})$, of separation length. In turn, expecting that the minimal size of penetrating length is $\mathcal{O}(L_e)$, the average value $\bar{Q} \approx \int_{1/Z}^1 Q^{-0.35} dQ / \int_{1/Z}^1 Q^{-1.35} dQ \approx 0.54 Z^{-0.35}$ is consistent with the power-law behavior reported in Fig. 1(B). Small, systematic differences towards $Q \rightarrow 1$ between $p(Q)$'s for rings with $Z = 114$ and $Z = 115$ should be attributed to incomplete equilibration of the corresponding data sets (see discussion in Sec. IB in SI). As explained

in the following, such deviations from the equilibrium distribution emerge also for smaller Z 's whenever polymer chains are not fully equilibrated. In sharp contrast with the results for equilibrated rings, $p(Q)$ distributions for rings constructed according to the IBP-model decay very differently, Fig. 1(D). This is particularly evident for very large rings, whose $p(Q)$'s feature an exponential cut-off towards $Q \rightarrow 1$. As the fine structure of the IBP-rings is, by construction, relaxed only up to spatial scales of the order of $Z \approx 1$ (Sec. IA in SI), we suspect threadings between large rings have not yet relaxed. Consistent with that, very short IBP-rings ($Z = 1.5$) are instead fully relaxed, as the corresponding $p(Q)$ exhibits the same universal equilibrium form from Fig. 1(C).

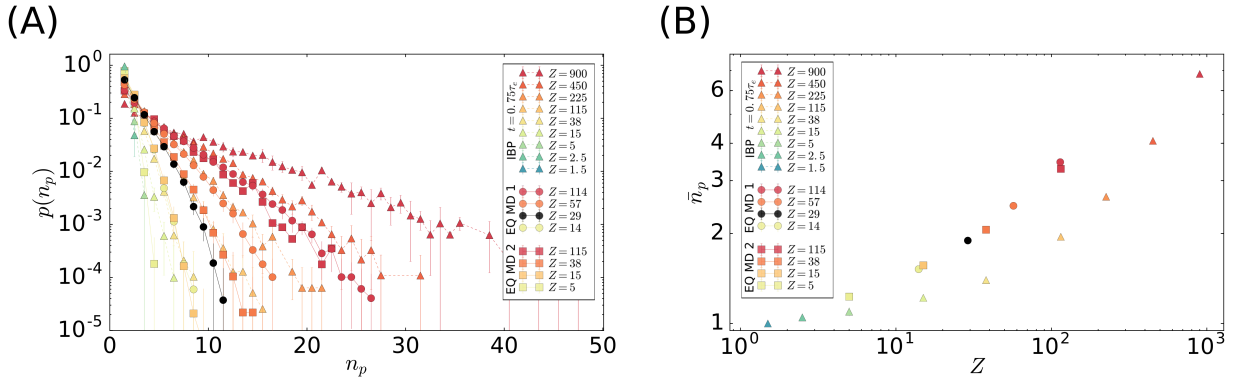


Figure 2: Threading statistics in terms of number of penetrations. (A) Probability distribution functions, $p(n_p)$, of the number of penetrations, n_p , for the different polymer models and ring masses Z (linear-log scale). (B) Corresponding mean number of penetrations, \bar{n}_p , as a function of the ring mass, Z (log-log scale).

We complete the discussion by focusing on how many times (n_p) any ring penetrates the minimal surface of any other single ring. In order to dismiss any fine scale detail related to the employed polymer model, a given threading segment contributes to n_p only if its contour length exceeds the entanglement length L_e .³⁸ We notice though, that with this constraint n_p is not necessarily an even number as in the original work.³³ Fig. 2(A) shows that distribution functions $p(n_p)$'s display exponential tails for both, MD-equilibrated rings (in agreement with³³) and IBP-model rings. Instead, corresponding *mean* values $\bar{n}_p \equiv \int n_p p(n_p) dn_p$ as functions of ring mass Z behave differently for MD-equilibrated *vs.* IBP-rings (Fig. 2(B)). As

for $\bar{Q}(Z)$, $\bar{n}_p(Z)$ from different MD simulations nicely collapse on a single curve. However, at odds with $\bar{Q}(Z)$ (Fig. 1(B)), $\bar{n}_p(Z)$ is laying at the threshold of a (slow) crossover and, consequently, our attempt to fit the data for $Z \geq 29$ to a single power-law behavior gave poor results. Obviously, the lower values for $\bar{n}_p(Z)$ from non-equilibrated rings reflect, as for $\bar{Q}(Z)$, how these chains systematically “underthread” their spatially-close neighbors. Due to the exponential character of the $p(n_p)$ distributions and the fact that each ring threads its neighbors (see Sec. IE in SI), the mean value \bar{n}_p is a good indicator of the typical number of penetrations made by a single ring.

Threadings dynamics – We are now going to discuss how almost-unthreaded rings constructed according to the IBP-model progressively thread each other. These rings reproduce several properties of equilibrated ring conformations like the gyration radius and contact probabilities.¹¹ On the other hand (Figs. 1 and 2), they fail in reproducing threading statistics. Therefore, we track how threading statistics is changing as ring conformations are relaxing over time. In the following, time is always expressed in units of the entanglement time τ_e ,²⁹ corresponding to the characteristic time-scale above which entanglements start slowing down chain dynamics.

Fig. 3(A) shows the evolution of the distribution function $p(Q, t)$ for $Z = 115$ (similar plots are obtained for $Z = 5, 15$ and 38 , not shown) at different times. For short times, the distribution $p(Q, t)$ is a power-law for $Q \rightarrow 0$ and has an exponential cutoff at larger $Q \rightarrow 1$, as in Fig. 1(D). As time increases, the exponential cutoff is progressively shifting to larger Q values as longer threadings occur. Then, we consider how the mean value, $\bar{Q}(t) \equiv \int Q p(Q, t) dQ$, changes with time (Fig. 3(B)). Interestingly, $\bar{Q}(t)$ grows at early times according to the simple power-law:

$$\bar{Q}(t) \sim (t/\tau_e)^{\alpha_Q}. \quad (2)$$

For $Z = 5, 15, 38$ this regime is followed by a plateau implying that equilibrium has been

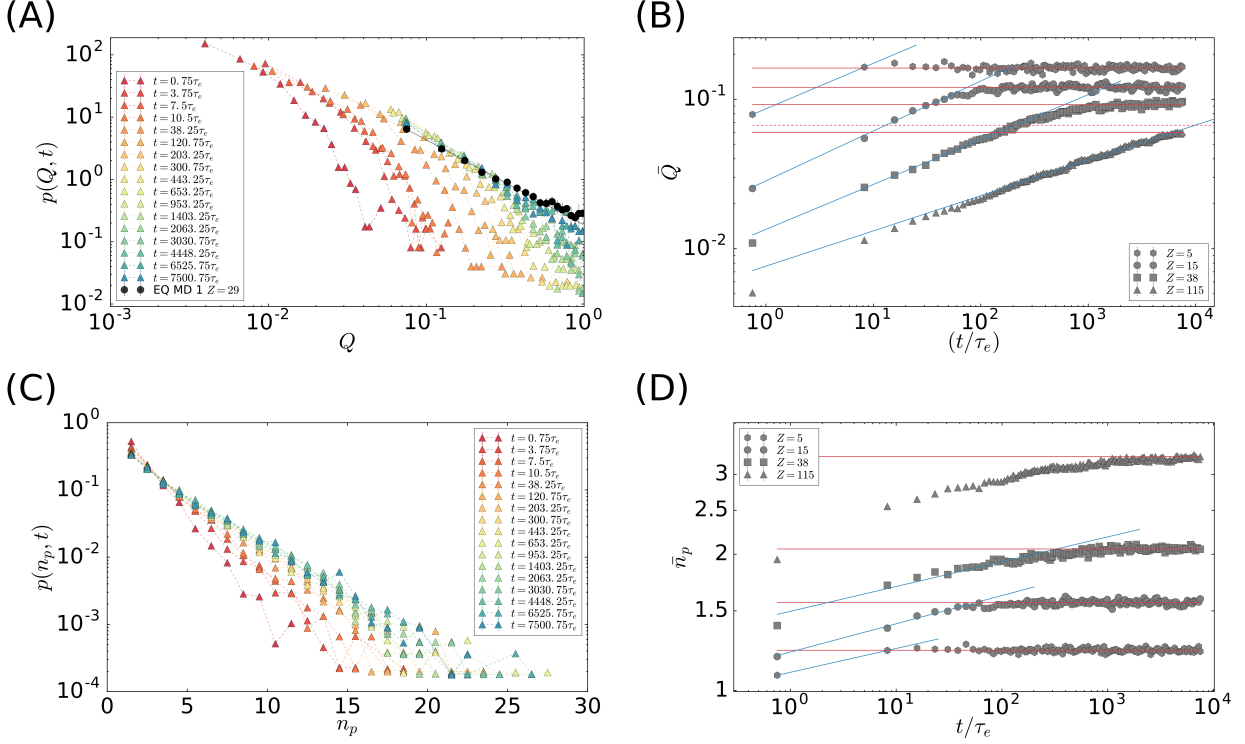


Figure 3: Time evolution of threading statistics. (A) Time-dependent distribution functions, $p(Q, t)$, of the relative contour length fraction, Q , of one ring threading another ring (log-log scale). Results for solutions of rings with $Z = 115$ prepared according to the IBP-model. Similar curves are found also for other Z 's (not shown). Black circles represent the equilibrium distribution $p(Q)$ calculated for rings with $Z = 29$ (Fig. 1(C)). (B) Corresponding mean values, $\bar{Q}(t)$ (symbols), as functions of time (log-log scale) and power-law fits to the data (Eq. (2), blue lines) in the initial stage of the equilibration. Solid horizontal lines for $Z = 5, 15, 38$ denote corresponding equilibrium values $\bar{Q}(Z)$. For $Z = 115$, the solid line is for the value measured at the end of the trajectory and the dashed line is for the extrapolated equilibrium value. (C) Time-dependent distribution functions, $p(n_p, t)$, of the number of penetrations, n_p (linear-log scale). Similar curves are found also for other Z 's (not shown). (D) Corresponding mean number of penetrations, $\bar{n}_p(t)$ (symbols), as functions of time (log-log scale) and power-law fits to the data (Eq. (3), blue lines) in the initial stage of the equilibration. Horizontal lines are for asymptotic values $\bar{n}_p(Z)$.

reached. Best fits of Eq. (2) to the data before the plateau (blue lines in Fig. 3(B)) give effective exponents $\alpha_Q \approx 0.3$ (for specific values, see Table S2 in SI). The heights of the different plateaus correspond (solid horizontal lines) to the equilibrium values for $\bar{Q}(Z)$ (symbols “□” in Fig. 1(B)). For $Z = 115$ instead, due to the incomplete equilibration, threadings are still evolving. In this case, the height of the corresponding plateau (dashed horizontal line) is *extrapolated* from the reported (Fig. 1(B)) power-law behavior $\bar{Q}(Z) \approx 0.26Z^{-0.31}$. The intercept between the fitted power-law and the plateau defines the threading relaxation time, $\tau_{\text{rel},Q}^{\text{th}}(Z)$ (for specific values, see Table S2 in SI). Interestingly, $\tau_{\text{rel},Q}^{\text{th}}(Z)$ is of the same order of the relaxation times, $\tau_{\text{rel}}^{\text{diff}}(Z)$, associated to ring thermal diffusion (Table S2 in SI) and defined (see Sec. IB in SI) at the intercept between the time mean-square displacement of the ring center of mass, $\langle g_3(t) \rangle \equiv \langle (\vec{r}_{cm}(t) - \vec{r}_{cm}(0))^2 \rangle$ and the time dependent mean-square gyration radius, $\langle R_g^2(t) \rangle$. On the other hand, $\tau_{\text{rel}}^{\text{diff}}$ is expected⁸ to be significantly larger than the time-scale associated to *internal* ring motion,⁸ $\tau_{\text{rel}}^{\text{int}} \equiv \int_0^\infty \frac{\langle \vec{c}(t) \cdot \vec{c}(0) \rangle}{\langle c(0)^2 \rangle} dt$, where $\vec{c}(t) = \vec{d}_1(t) \times \vec{d}_2(t)$ and $\vec{d}_1(t)$ and $\vec{d}_2(t)$ are any arbitrarily chosen pair of spanning ring diameters whose tails are separated by the contour length $Z/4$. Accurate numerical evaluation of $\tau_{\text{rel}}^{\text{int}}$ (see Sec. IB in SI) confirms that $\tau_{\text{rel}}^{\text{int}} < \tau_{\text{rel}}^{\text{diff}}$ at any given Z (Table S2 in SI). Threadings constitute then the dominant degrees of freedom governing rings relaxation.

We complete our analysis by considering the time evolution of the distribution function of the number of penetrations, $p(n_p, t)$, as the rings progressively thread (Fig. 3(C)) as well as the corresponding average value, $\bar{n}_p(t) \equiv \int n_p p(n_p, t) dn_p$ (Fig. 3(D)). Data appear slightly noisier than the ones for $\bar{Q}(t)$ (Fig. 3(B)), yet $\bar{n}_p(t)$ is also clearly exhibiting an initial power-law regime:

$$\bar{n}_p(t) \sim (t/\tau_e)^{\alpha_{n_p}}, \quad (3)$$

followed by given plateaus for $Z = 5, 15, 38$ whose heights (solid horizontal lines) correspond to the equilibrium values $\bar{n}_p(Z)$ (symbols “□” in Fig. 2(B)). In those cases, the effective exponents α_{n_p} are close to ≈ 0.06 while the crossover times $\tau_{\text{rel},n_p}^{\text{th}}(Z)$ match well corresponding

$\tau_{\text{rel},Q}^{\text{th}}(Z)$'s (Table S2 in SI). As for $Z = 115$, arguably because of incomplete equilibration, the initial crossover to equilibrium resembles less a single power-law compared to the other cases with smaller Z . Since the evaluation of the asymptotic behavior at large t is also problematic (see discussion on threading statistics), corresponding α_{n_p} and $\tau_{\text{rel},n_p}^{\text{th}}$ can not be reliably estimated.

Conclusions – Theoretical considerations^{1,3,31} corroborated by recent numerical work¹¹ led to the conclusion that topologically constrained ring polymers like rings in a gel³⁶ or rings in concentrated solutions and melt^{7,8,11,12} should resemble double-folded conformations with randomly branched structures.

In this Letter, we have shown that this picture is not complete as it tends to underestimate the correct extent of threadings^{17,20,34,35} between close-by rings at equilibrium. Following,^{10,33} our analysis relies upon the concept of ring minimal surface and our results are independent from model details: in particular we report that both, the relative contour length penetrating the minimal surface of a given ring (Q and its distribution $p(Q)$, Fig. 1) and the absolute number of penetrations (n_p and its distribution $p(n_p)$, Fig. 2) display universal features. At the same time, we have demonstrated that threading relaxation to equilibrium (functions $\bar{Q}(t)$ and $\bar{n}_p(t)$, Fig. 3) is power-law, and that the associated time-scales match ring diffusion in melt while remaining significantly larger than the time-scales associated to ring internal relaxation. Based on that, we predict that threadings dominate ring relaxation in entangled solutions. At the same time, two of our results also hint on the reason why double-folded models work well:¹¹ (1) the observed relation $\bar{Q}(Z)^{-1} = (Z - Z_{\text{sep}})/Z_{\text{sep}} \sim Z^{0.31}$ implying that the separation length $Z_{\text{sep}} \equiv L_{\text{sep}}/L_e$ increases only sublinearly in the ring mass Z and (2) the small (Fig. 2(B)) mean number of threadings. The static properties could then be well governed by the larger unthreaded contour length $L_c - L_{\text{sep}}$, in agreement with the tree picture. Yet, the smaller L_{sep} could affect the dynamics.

We speculate that the exponent $\alpha_Q \approx 0.3$ governing threading relaxation could be (related to) the exponent $1/3$ of the late-stage phase-ordering kinetics with a conserved order

parameter.⁴⁰⁻⁴² If the number of branches of the ring conformation is conserved during the relaxation from the IBP state, the curvilinear diffusion of the branches could be viewed as switching the branches from a non-threading to a threading state. To find out if the correspondence does exist, we would need to connect our threading analysis with an algorithm to detect branches such as the one in Ref.¹⁸

A limitation of the present analysis is that while concentrating primarily on pairwise threadings, it neglects higher-order ones whose contribution to ring dynamics in melts appears to be not negligible³⁵. In future, a potential non-invasive method to detect the complex threadings could help to clarify their microscopic origin and effect.

At the light of these results, the question related to how to construct “by first principles” equilibrated solutions of ring polymers not based on double-folded conformations¹¹ is still open: whether the answer will require to rethink double-folded conformations or a completely different approach, in both cases it remains a promising research line for the future.

Acknowledgement

We thank Alexander Grosberg and Michael Lang for useful discussions. JS acknowledges support from the Austrian Science Fund (FWF) through the Lise-Meitner Fellowship No. M 2470-N28. The authors would like to acknowledge networking support by the COST Action CA17139.

Supporting Information Available

The following files are available free of charge.

- The file contains additional details on: preparation of initial melts of ring conformations, Molecular Dynamics computer simulations and the algorithm to compute minimal surfaces.

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